

# Notes & Comments

## ON CAPITAL ALLOCATION AND THE REAL COST OF LABOUR

by

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This note is an extension of several contributions to the problem of resource allocation in a developing economy. In separate papers, I.M.D. Little and F. Seton<sup>1</sup> have introduced a model in which labour in a developing economy *cannot* be shifted from the subsistence to the industrial sector at zero opportunity cost, even though this labour displays zero marginal product in its 'traditional' occupations; and in what follows this problem will be attacked via a diagrammatic analysis. A short appendix will treat a side issue of the topic.

As Little presented the model, there was an initial amount of capital  $\bar{K}$  to be divided between two sectors, the I (industrial) sector, and the C (subsistence, traditional, or agricultural) sector. In the C-sector, there is excess labour or disguised unemployment, in the sense of Professor W. A. Lewis<sup>2</sup>, in that the marginal product of labour in this sector is taken as equal to zero. As it happens, however, this labour cannot be moved to the I-Sector without an increase in production in the C-sector. The reason for this is because as labour is transferred to the industrial sector, consumption per head increases in the C-sector, thus decreasing the surplus available for workers being transferred to the I-sector. The transfer can only be carried out if a surplus equal to the difference between the industrial wage in C-goods and the amount of C-goods 'released' by the C-sector is forthcoming, and for this an increased production of C-goods (via the input of capital into the C-sector) must take place. A similar situation would exist if transferring workers required a wage differential; or if C-goods had to be exported to obtain certain types of capital goods for the labour being reallocated, and/or housing, training, *etc.*

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<sup>1</sup>The key references are I.M.D. Little [7] and F. Seton [8]. Subsequent analyses are many, but prominent amongst these are Ronald Findlay [2], L. Lefebvre [4], and Ramgopal Agarwala [1].

<sup>2</sup>All literature on disguised unemployment usually begin with W.A. Lewis [5].

As mentioned, the initial capital available is  $\bar{K}$ . Assume a simple linear production function in the C-sector of  $q_c = q_c(K_c) = \lambda K_c$ , and a linear homogeneous production function  $q_i = \bar{q}_i(K_i, L_i)$  in the I-sector. Then, from the condition that the amount of labour that can be transferred is a function of the amount of surplus available, we can get  $L_i = L_i(K_c)$ , and it becomes possible to write  $q_i = q_i(K_i, K_c)$ <sup>3</sup>. Ignoring income existing prior to the appearance of  $\bar{K}$ , a graphical presentation of this situation can be made (Figure 1).

In the third quadrant of Figure 1, the production function for  $q_i$  is shown along with capital  $\bar{K}$ . Beginning from A, where  $q_i$  is zero, and moving toward B, successively higher isoquants of  $q_i$  are crossed, until we come to B, where  $q_i$  is a maximum and equal to  $q_0$ . Continuing in this direction,  $q_i$  begins to decrease, becoming zero again at C'. As for  $q_c$ , this is zero at A (where  $K_c = 0$ ) and goes to a maximum at C" (where all capital is being used to produce the C-good). The 'transformation' curve is shown in the first quadrant.

There is then the matter of choice insofar as the ( $q_c, q_i$ ) mix is concerned; and this, in reality, amounts to a choice between present and future goods — the assumption being that incremental activity in the industrial sector only involves the producing of more capital goods. The first step here, however, consists of clarifying just 'whose' choice we are talking about. If there is just one scarce good, capital, then all value must be imputed to the owner of that good; and thus, the choice must necessarily be that of the owners of capital. This specification, of course, serves only for a theoretical paper; however, a reasonable variant of this argument might be that all capital-owners — to include those performing other functions — could be represented in the preference function, with their weight in this function being proportional to their holding of capital. Thus, the government could also be represented — either directly through its consumption or investment of tax returns, or indirectly as, e.g., through the behaviour of the recipients of government transfers<sup>4</sup>.

Continuing, we have the arrangement at B (with  $q_0$  the relevant isoquant) which corresponds to B', where the production of  $q_i$  is a maximum. Attaining this position, however, poses problems that in the real world might seem insurmountable: at B (and B') total capital  $\bar{K}$  is employed either directly as an input in the I-sector or transformed into industrial labour; and the consumption of capital-owners is zero. My own examination of the literature, however, leads me to believe that savings ratios of 0.15-0.20 are the best that one can hope

<sup>3</sup>To follow this sequence, assume, in addition to the information already given, that it requires  $w$  in "additional" production of C-goods to transfer one worker, where  $w$  (like  $\lambda$ ) is exogenously established. Thus, we have  $q_c = q_c(K_c) = \lambda K_c$ , and  $L_i = \lambda K_c / w$  as the number of workers that  $K_c$  of capital can transfer. Then, with  $q_i = q_i(K_i, L_i)$ , we substitute for  $L_i$  and get  $q_i = q_i(K_i, K_c)$ .

<sup>4</sup>I should like to acknowledge the assistance of a referee for pointing out certain omissions and shortcomings in my treatment of the topics of this paragraph in an earlier draft.

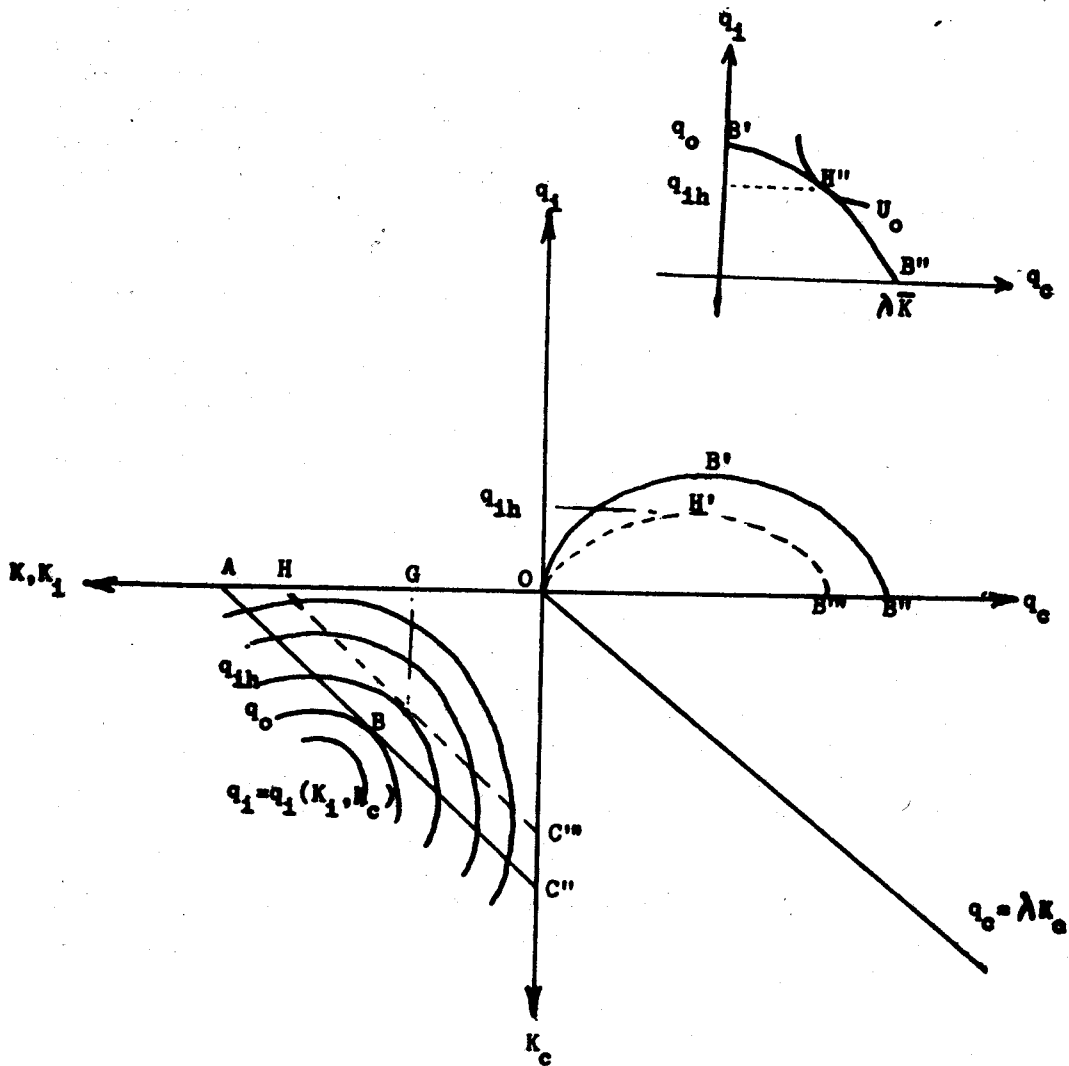


Figure 1

for in these circumstances, and, thus, the significance of a point, such as B', is largely theoretical.

The question of more realistic savings ratios will be taken up below, but first it should be mentioned that point B' is regarded by Little as the 'optimum optimorum' in this type of model: "The point where one cannot have more investment as well as more consumption". Seton more or less explicitly refers to B' as "...optimal in the sense of yielding the greatest possible addition to capital, and therefore maximum growth potential for the future". It is also clear that at B', on the basis of the 'isoquant-isocost' tangency at the corresponding point B, we have:

$$\frac{\partial q_1 / \partial K_i}{\partial q_1 / \partial K_c} = 1 \dots \dots \dots (1)$$

This condition, and its equivalent, will be derived in the appendix from the dual of a nonlinear programme; but it should be obvious, since the slope of the 'isocost' AC'' is unity. Moreover, as a maximising condition, this expression seems quite reasonable. What it says is that the incremental unit of capital should be added to the sector where its addition causes the largest increase in the output of q<sub>1</sub>. If, e.g., we have ∂q<sub>1</sub>/∂K<sub>i</sub> > q<sub>1</sub>/∂K<sub>c</sub>, then capital should be taken away from the C-sector and added to the I-sector.

In connection with developing an expression for the rate of growth corresponding to a condition of the type presented in (1), it should be appreciated that while at B' we have consumption, it should not be considered final consumption. As Lefebvre points out, its purpose is to secure industrial labour and, thus, it should be classified as an intermediate good: it would not appear among those components of final demand that form national income. Given this, we see that capital  $\bar{K}$  is, in general, divided into three parts: K<sub>i</sub>, the industrial input; K<sub>c</sub>, the intermediate good of the type mentioned above; and K'<sub>c</sub>, the consumption of capital-owners. We, thus, have K = K<sub>i</sub> + K<sub>c</sub> + K'<sub>c</sub>, with K'<sub>c</sub> = (1-s)  $\bar{K}$ , or s $\bar{K}$  = K<sub>i</sub> + K'<sub>c</sub>, where s is the savings coefficient. Again, writing the (homogeneous) production function in the I-sector as q<sub>1</sub> = q<sub>1</sub>(K<sub>i</sub>, K<sub>c</sub>), we get from Eulers' theorem:

$$q_1 = f_K K_i + f_K K_c = f_K (K_i + K_c) \dots \dots \dots (2)$$

where f<sub>K</sub> is the marginal productivity of capital. This can be written:

$$q_1 = f_K s \bar{K} = d\bar{K} \dots \dots \dots (3)$$

since q<sub>1</sub> = d $\bar{K}$  in the increase in total capital  $\bar{K}$ . We can, thus, write:

$$g_K = \frac{dk}{\bar{K}} = s f_K \dots \dots \dots (4)$$

At point B', we have  $s=1$ , and, thus,  $fK = gK = gK_{\max}$ . The question can now be raised as to what happens when  $s \neq 1$ .

In Figure 1, when we are not at point B, and all capital  $\bar{K}$  is not being used as an input in the I-sector or as an intermediate good, we have consumption by the owners of capital. Take as an example the point H, which signified that the capital being transformed into the consumption of capital-owners is HA, and which, when multiplied by  $\lambda$  (the transformation rate of capital into C-goods), gives the value of this consumption in C-goods.

If we now draw a new 'isocost' HC''' and repeat the exercise described earlier, we get a new transformation curve OB'''. The optimum here is at H', with  $q_{ih}$  the output of I-goods, and B''B''' the consumption of capital-owners measured in C-goods. We also see that capital input in the I-sector is OG, while GH is the amount being used for intermediate consumption. As for the algebra of this situation, we have  $0 < s < 1$ , with  $K'_c \neq 0$ . In terms of Figure 1, we have  $\bar{K} = K_i + K_c + K'_c = OG + GH + HA$  and, thus,  $s\bar{K} = OH$ . In expression (4) we then have  $g_K = sf_K < g_{K_{\max}}$

A consumption possibility curve for capital-owners is now a relevant topic. This takes the appearance of a conventional transformation curve, and is shown in a mini-diagram in the first quadrant of Figure 1. The position of maximum investment (corresponding to point B' and  $q_0$ ) is again given by B'. On the other hand, the position of no investment (and maximum consumption by capital-owners) is given by B''. As an example of an intermediate position, point H' and a representative preference curve is shown. This obviously corresponds with points H' and H, and isoquant  $q_{ih}$ .

The final topic treated here will concern the effect of the productivity of capital on investment. This aspect of the problem has not been considered at all in the papers referred to above; and in the context of the present type of model has been taken up formally only by Professor Leontief and, in a recent paper, by Ashok Guha<sup>5</sup>.

Given the consumption possibility curve, shown in Figure 2-a, a simplified approach to this problem involves examining the return to capital under alternative assumptions about productivity. We know, for example, that one measure of this return to capital is the stream of future consumption available if we refrain from one unit of C-goods today. Rather than make a direct calculation of this return, it might be interesting to compare transformation curves  $T_1$  and  $T_2$  in Figure 2-a.

If  $dC$  in present consumption is given up, it means that the increment  $dK_1$  can be invested if  $T_1$  is the curve in question, or  $dK_2$  if it is  $T_2$ . Assuming, for

<sup>5</sup>See W. Leontief [6] and Ashok Guha [3].

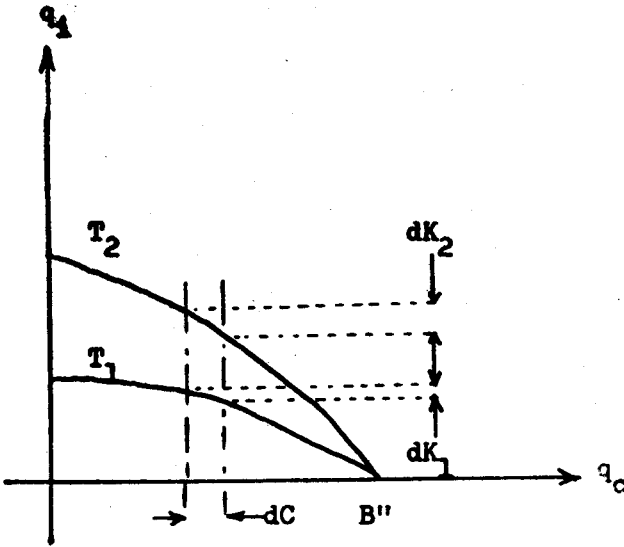


Figure 2-a

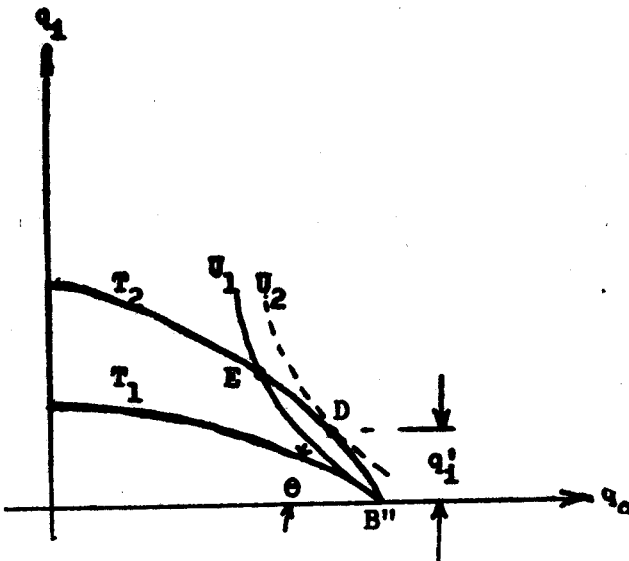


Figure 2-b

the sake of simplicity, that in the next period this increment is turned into consumption by taking it to the consumption sector and combining it with the free labour there to produce  $\lambda dK$  of C-goods until eternity, it seems obvious that where the discounted value of this return is concerned we, have:

$$\int_{t_1}^{\infty} dK_2 e^{-\eta t} dt > \int_{t_2}^{\infty} dK_1 e^{-\eta t} dt \dots \dots \dots (5)$$

regardless of what we use for the discounting factor  $\eta$ . The question then becomes why should  $T_2$  be outside of  $T_1$ . Considering only those elements that are found in this model, two reasons can be cited: *i*) a lower real price of labour in the case of  $T_2$  ( $w_2 < w_1$ ); and *ii*) a higher productivity of capital in the industrial sector in the case of  $T_2$ . A higher productivity of capital in the C-sector is also a possibility here, but this case is more complicated since it also involves a displacement of the transformation curve to the right.

To emphasise these and other points, a simple example will be constructed around Figure 2-b. First of all it should be realised that just as point  $B'$  represents a situation where the social value of consumption is equal to zero, we have at  $B''$  a situation where the weight attached to investment is zero.

We then designate a value for the slope of  $T_1$  at  $B''$ , and this will be called  $\theta$ . Then take as the slope of all preference curves  $\rho = \theta + e$ , where  $e$  is 'very' small and non-negative. Such a specification, thus, results in a situation of the sort mentioned in the previous paragraph where there is no investment. The relevant preference curve here is  $U_1$ .

But now ask what would be the situation if there was an increase in productivity that sent  $T_1$  into  $T_2$ . Point  $B''$  no longer represents an equilibrium, and  $U_1$  intersects the transformation curve at E. This arrangement, however, signifies a higher rate of return on capital (measured in C-goods) than the rate of time preference; and so equilibrium is displaced to D (where investment is  $q_1$ ) and we are on preference curve  $U_2$ .

If the productivity of capital gives us a transformation curve of the type  $T_1$ , it would appear that only a community with a very low time preference would do any saving at all. This, it appears, is one of the more important problems addressing itself to development economists today; but while simple theoretical models may perhaps suffice where a presentation of these matters is concerned, solutions will be considerably harder to come by.

# Appendix

In a paper cited above, Mr. R. Agarwala has provided a very lucid, but not entirely fault, free analysis of some of the issues discussed in this paper. With  $\lambda$ , the productivity of capital in the C-sector, and  $w$  the 'additional' consumption necessary to release a worker to the I-sector, Agarwala has presented, without proof, the expression:

$$\frac{\partial q_i / \partial L_i}{\partial q_i / \partial K_i} = \frac{w}{\lambda} \dots \dots \dots (6)$$

as the ratio of the shadow price of labour to capital when  $q_i$  is maximised. I should now like to derive this equation as one of a class of optimal allocation problems, and at the same time demonstrate that it is formally equivalent to (1) above. Remembering that the amount of labour  $L_i$  that can be transferred by an amount of capital  $K_c$  is  $\lambda K_c / w$ , we can go immediately to the nonlinear programme:

$$\left. \begin{array}{l} \text{Max: } F(K_i, L_i) = F\left(K_i, \frac{\lambda K_c}{w}\right) \\ \text{subject to } s\bar{K} = K_i + K_c = K_i + \frac{wL_i}{\lambda} \\ \text{and, } K_i, K_c \geq 0. \end{array} \right\} \dots \dots \dots (7)$$

Since  $F$  is concave, we get immediately from the dual of (7)

$$\emptyset = \frac{\partial F}{\partial K_i} = \frac{\lambda}{w} \frac{\partial F}{\partial L_i} \quad (\text{or}) \quad \frac{\partial F / \partial K_i}{\partial F / \partial L_i} = \frac{\lambda}{w} \dots \dots \dots (8)$$

and

$$s\bar{K} = K_i + K_c$$

If we now notice that  $F(K_i, L_i)$  can be written  $F(K_i, K_c)$ , we can also get from the dual of (7):

$$\frac{\partial F / \partial K_i}{\partial F / \partial K_c} = 1 \dots \dots \dots (1)$$



We have, thus, succeeded in deriving Agarwala's expression, but its interpretation, viewed in the light of the model discussed in this paper, must be somewhat different;  $w$  is the *real* price of labour, measured in consumption goods; while  $w/\lambda$  is the same price measured in industrial goods. Alternatively, the inverse of this expression gives the transformation rate of capital into industrial labour; and in its capacity as an optimising condition, it has a direct correspondence with points B and B' in Figure 1.

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